2022.M30

2022L003A2EL

Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination 2022

Mathematics

Paper 2

Higher Level

Monday 13 June   Morning 9:30 - 12:00

220 marks

Examination Number

Day and Month of Birth

For example, 3rd February is entered as 0302

Centre Stamp

The 2022 examination papers were adjusted to compensate for disruptions to learning due to COVID-19. This examination paper does not necessarily reflect the same structure and format as the examination papers of past or subsequent years.

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Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	120 marks	6 questions
Section B	Contexts and Applications	100 marks	4 questions

Answer questions as follows:

- any **four** questions from Section A – Concepts and Skills
- any **two** questions from Section B – Contexts and Applications.

Write your Examination Number in the box on the front cover.

Write your answers in blue or black pen. You may use pencil in graphs and diagrams only.

This examination booklet will be scanned and your work will be presented to an examiner on screen. Anything that you write outside of the answer areas may not be seen by the examiner.

Write all answers into this booklet. There is space for extra work at the back of the booklet. If you need to use it, label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You will lose marks if your solutions do not include relevant supporting work.

You may lose marks if the appropriate units of measurement are not included, where relevant.

You may lose marks if your answers are not given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Section A	Concepts and Skills	120 marks

Answer **any four questions** from this section.

Question 1 **(30 marks)**

(a) The table below gives some details on the number of different types of student in a university. There are 22 714 students in the university in total.

	Age (years)		**Total**
	23 or younger	**24 or older**	
Undergraduate	12 785	2922	**15 707**
Postgraduate	1353		
Total		**8576**	**22 714**

(i) Fill in the three missing values to complete the table above.

(ii) One student is picked at random from the students in the university.
Let **O** be the event that the student is 24 years old, or older.
Let **U** be the event that the student is an undergraduate.

Are the events **O** and **U** independent? Justify your answer.

Conclusion: ____

(b) Three people are picked at random from a class.
Find the probability that all three were born on the same day of the week.
Assume that the probability of being born on each day is the same.

(c) There are b boys and g girls in a class, where $b, g \in \mathbb{N}$.
$\frac{3}{5}$ of the students in the class are girls.

4 boys and 4 girls join the class.
One student is then picked at random from the whole class.
The probability that this student is a girl is now $\frac{4}{7}$.

Find the value of b and the value of g.

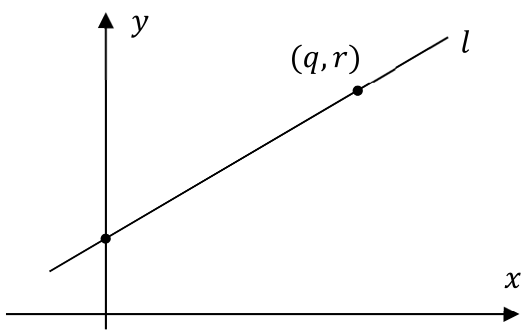
$b =$ ____________________ $g =$ ____________________

Question 2 **(30 marks)**

(a) The points $A\ (8, -4)$ and $B\ (-1, 3)$ are the endpoints of the line segment $[AB]$.

Find the coordinates of the point C, which divides $[AB]$ internally in the ratio $4 : 1$.

(b) The line l has a slope of m and contains the point (q, r), where $m,\ q,\ r \in \mathbb{R}$ are all positive.
Find the co-ordinates of the point where l cuts the y-axis, in terms of m, q, and r.



Answer: (,)

(c) The line k has a slope of -2.
The line j makes an angle of $30°$ with k.

Find **one** possible value of the slope of the line j.
Give your answer in the form $d + e\sqrt{f}$, where $d, e, f \in \mathbb{Z}$.

Question 3 **(30 marks)**

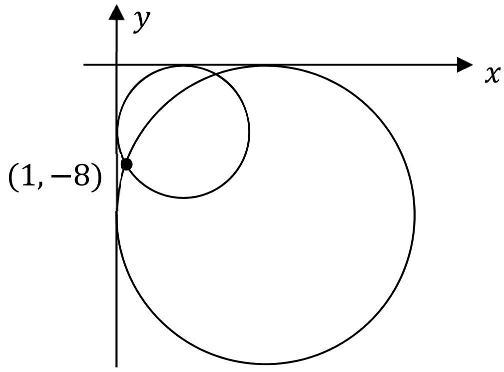
(a) The circle c has equation $x^2 + y^2 - 2x + 8y + k = 0$, where $k \in \mathbb{R}$.
The radius of c is $5\sqrt{3}$.

Find the value of k.

(b) The circle $(x-5)^2 + (y+2)^2 = 20$ has a tangent at the point $(9, -4)$.
Find the slope of this tangent.

(c) Two circles each have both the x-axis and the y-axis as tangents, and each contains the point $(1, -8)$, as shown in the diagram on the right (not to scale).

Find the equation of **each** of these circles.



Question 4 **(30 marks)**

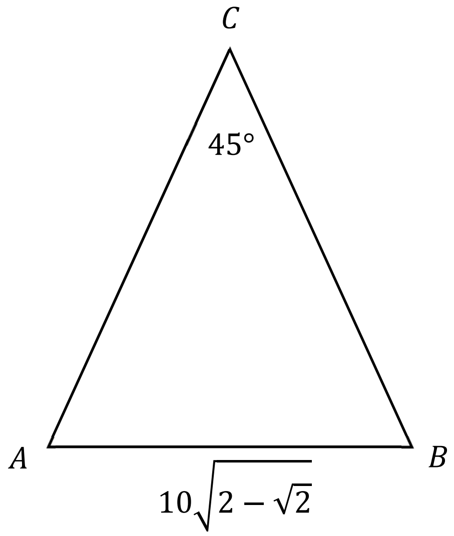
(a) **(i)** Prove that $\tan(A-B)=\dfrac{\tan A-\tan B}{1+\tan A\tan B}$.

(ii) Write $\tan 15^\circ$ in the form $\dfrac{\sqrt{a}-1}{\sqrt{a}+1}$, where $a\in\mathbb{N}$.

(b) The triangle ABC is shown in the diagram below.

$|AC| = |BC|$ and $|\angle ACB| = 45°$. $|AB| = 10\sqrt{2-\sqrt{2}}$, as shown.

Find the length $|AC|$.



Question 5 **(30 marks)**

(a) A survey on remote learning was carried out on a random sample of 400 students.
135 of the students preferred remote learning over in-person learning.
For **parts (a)(i)**, **(a)(ii)**, and **(a)(iii)**, give all solutions as decimals, correct to 4 decimal places.

(i) Work out the proportion of the sample that preferred remote learning.

(ii) Use the margin of error $\left(\frac{1}{\sqrt{n}}\right)$ to create a 95% confidence interval for the proportion of the population that preferred remote learning.

(iii) Using the proportion from **part (a)(i)**, create a 95% confidence interval for this population proportion that is more accurate than the 95% confidence interval based on the margin of error.

(b) In 2019, people with a pre-pay mobile phone plan spent an average (mean) of €$20 \cdot 79$ on their mobile phone each month (source: www.comreg.ie).

In 2021, some students carried out a survey to see if this figure had changed.
They surveyed a random sample of 500 people with pre-pay mobile phone plans.
For this sample, the mean amount spent per month was €$22 \cdot 16$ and the standard deviation was €$8 \cdot 12$.

Carry out a hypothesis test at the 5% level of significance to see if this shows a change in the mean monthly spend on mobile phones for people with a pre-pay plan.
State your null hypothesis and your alternative hypothesis, state your conclusion, and give a reason for your conclusion.

Null Hypothesis:

Alternative Hypothesis:

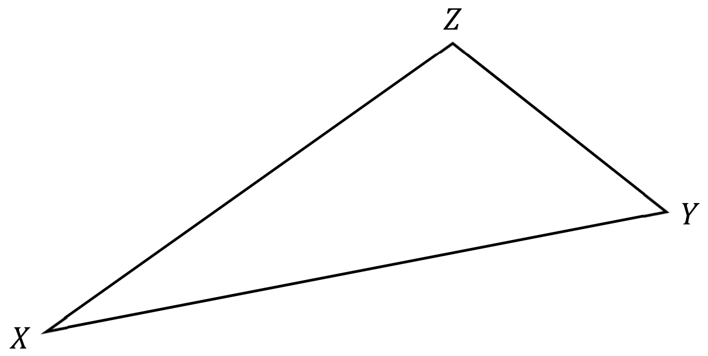
Calculations:

Conclusion:

Reason for your conclusion:

Question 6 **(30 marks)**

(a) Construct the **circumcentre** of the triangle XYZ shown below, using only a compass and straight edge. Label the circumcentre C. Show your construction lines clearly.

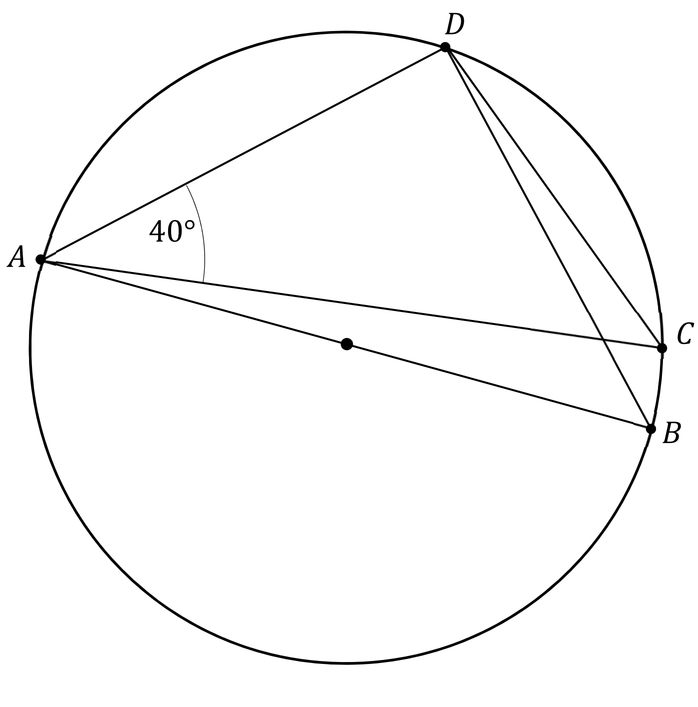


(b) The points A, B, C, and D lie on a circle, as shown in the diagram on the right (not to scale).

$[AB]$ is a diameter of the circle.
$|\angle DAC| = 40°$, as shown.
The triangle ABD is isosceles.

Find $|\angle ADC|$.

There is space for your solution on the next page.

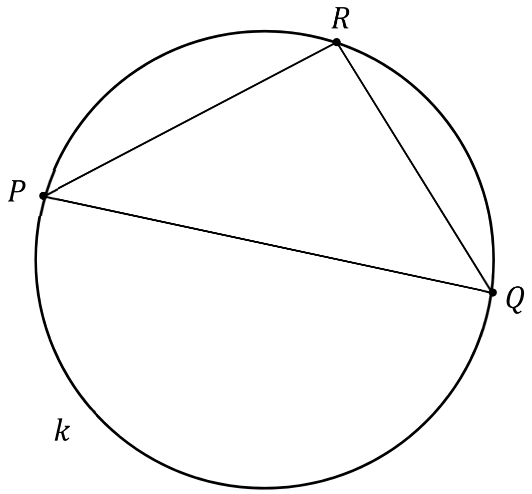


*Answer for **part (b)**:*

The diagram on the right shows the triangle PQR (not to scale).

The angle at R is **greater** than $90°$.

k is the circumcircle of PQR, as shown,
and O is the circumcentre.
O is not shown in the diagram.



(c) Prove that O **cannot** be inside the triangle PQR.

If you are proving this by contradiction, your first line should be:

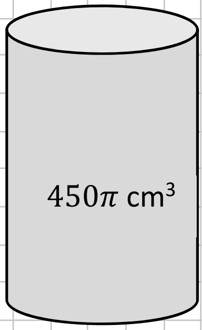
"Assume that O **is** inside the triangle PQR."

Answer **any two questions** from this section.

Question 7 **(50 marks)**

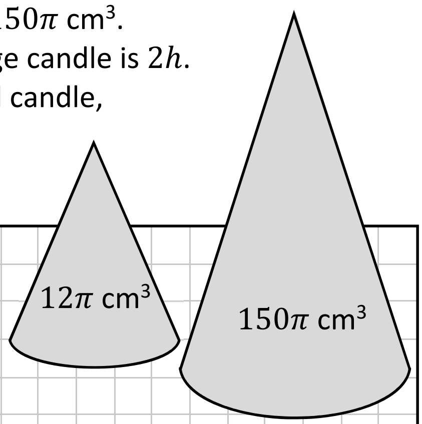
A company makes and sells candles of different shapes and sizes.

(a) A candle in the shape of a cylinder has a **diameter** of 10 cm and a volume of 450π cm^3.
Work out the height of this candle.



(b) A small candle in the shape of a **cone** has a volume of 12π cm^3.
A large candle, also in the shape of a cone, has a volume of 150π cm^3.
The height of the small candle is h, and the height of the large candle is $2h$.
The large candle has a radius that is k times that of the small candle, where $k \in \mathbb{R}$.

Work out the value of k.

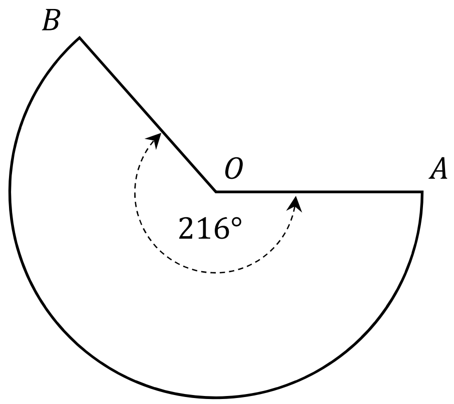


(c) A third conical candle has its curved surface area covered in cloth.

The net of the cloth is shown in the diagram below (not to scale).
This net covers the cone perfectly, with no overlapping material.

In the diagram, $|\angle BOA| = 216°$, as shown. $|OA| = 8$ cm.

Find the length of the arc from B to A, in terms of π, **and** hence find the radius of the cone.
Give both answers in cm.



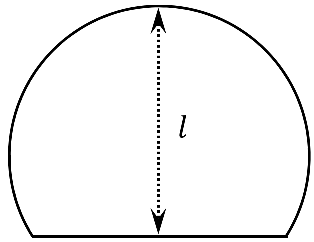
Length of arc from B to A: ____________________

Radius of cone: __________________

This question continues on the next page.

(d) **(i)** A spherical ball of wax is used as a candle. The radius of the sphere is $2 \cdot 7$ cm.
Find the volume of the sphere, correct to 3 decimal places.

(ii) A horizontal slice is cut off this sphere so that the candle will balance on level surfaces.



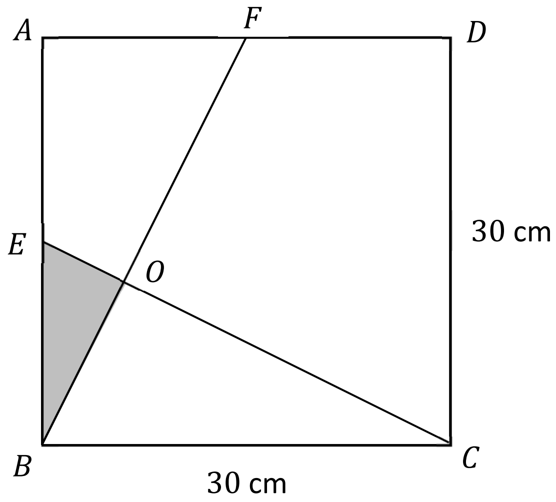
The area of the circular base of this candle is $5 \cdot 4$ cm^2.

Find the value of l, the vertical distance from the top of the candle to where the cut is made.
As shown in the diagram on the right, $l > 2 \cdot 7$ cm.

Give your answer in cm, correct to the nearest mm.

(e) Part of the logo of the company is shown below.
$ABCD$ is a square, with sides of length 30 cm.
The points E and F are the **midpoints** of $[AB]$ and $[AD]$, respectively.
The lines EC and FB are perpendicular, and meet at the point O.

Using similar triangles or trigonometry, find the length $|EO|$.
Give your answer in the form $a\sqrt{b}$ cm, where $a, b \in \mathbb{N}$.



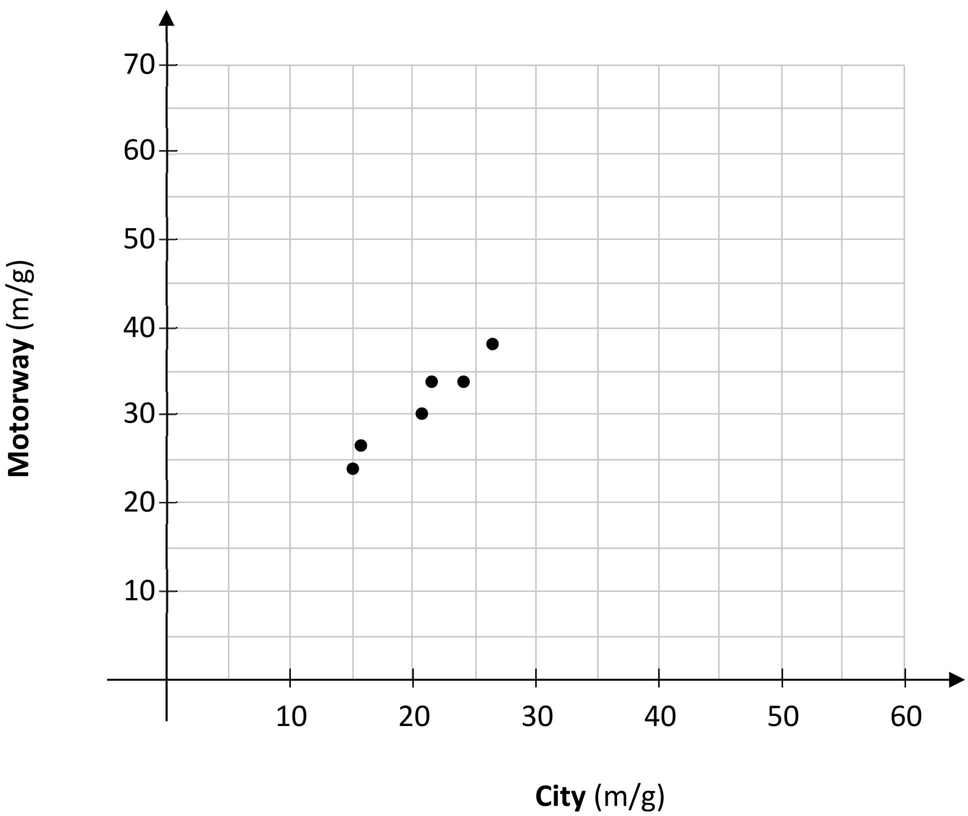
Question 8 **(50 marks)**

(a) Jena is researching fuel consumption in cars. She finds the following data for the number of miles per gallon (m/g) for eight different cars, labelled **A** to **H**, when driving in the city and on the motorway:

Miles per gallon data for city and motorway		
Car	**City** (m/g)	**Motorway** (m/g)
A	22	34
B	27	38
C	24	34
D	16	27
E	15	24
F	21	30
G	**30**	**40**
H	**17**	**30**

(i) The scatterplot below shows this data for cars **A** to **F**.

Using the data in the table above, **plot** and **label** points to represent cars **G** and **H** on the scatterplot below.



(ii) On the scatterplot, **draw** the line of best fit for the data, by eye.

(iii) Two other cars, **K** and **L**, have the miles per gallon values given in the following table.

Use your line of best fit on the scatterplot to fill in an estimate for each of the two missing values in the table below. Show your work on the scatterplot.

Car	**City** (m/g)	**Motorway** (m/g)
K	20	
L		60

(iv) Based on the data given, would you be more confident in the value you estimated for **K** or for **L**? Give a reason for your answer.

I would be more confident in my value for: **K** ☐ **L** ☐

(Tick (✓) **one** box only)

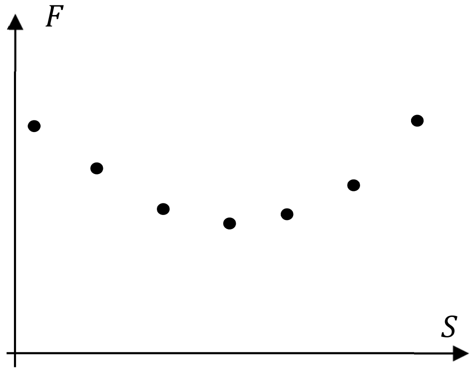
Reason:

(v) Find the value of r, the correlation coefficient between city and motorway miles per gallon. Use only the values for the 8 cars **A** to **H** in the table on the previous page. Give your answer correct to 3 decimal places.

This question continues on the next page.

(b) The scatterplot on the right shows some values of fuel consumption (F) for the given values of engine speed (S), for a particular car.
For the points in this scatterplot, F can be closely approximated by a quadratic function of S.

r_{FS} is the correlation coefficient between F and S, based on the points in this scatterplot.
Give a reason why you might think that r_{FS} is very close to 0.



(c) 13 customers rated their experience in a garage, by giving a whole-number score out of 100.
The mean score was 52. The median score was 54. No two scores were the same.

The table below shows the score for each of the 13 customers (in no particular order).
Stephen gave a score of **S** and Mary gave a score of **M**, where **S**, **M** $\in \mathbb{N}$.
Find the least value **and** the greatest value that **S** could be.

46	68	24	74	42	30	61	54	28	50	57	**S**	**M**

Least value of **S**: ____________ Greatest value of **S**: ____________

(d) John bought a car a number of years ago. The table below gives an estimate of the probability that each of the following three events happens to John's car in the next year.

Event	Probability
Head gasket blows	$0{\cdot}095$
Timing belt goes	$0{\cdot}041$
Air filters break	$0{\cdot}073$

(i) If the head gasket blows, John will have to replace his car, at an estimated cost of €$20\,000$. If the head gasket is replaced now, it will cost €1450, and the probability that it blows in the next year will be reduced to $0{\cdot}005$.

Based on these figures, use expected values to work out if it is worth replacing the head gasket now, or not.

Working out:

Conclusion:

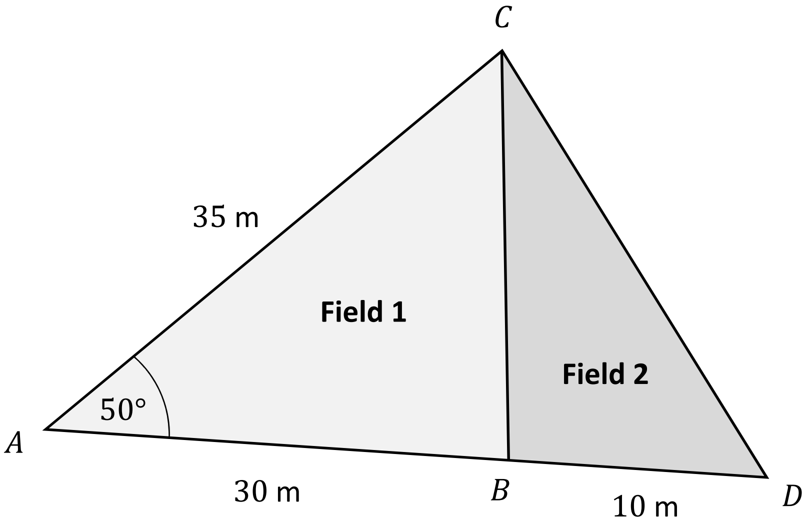
(ii) Work out the probability that **at least one** of the events in the table above happens to John's car this year, taking these events to be independent. Give your answer correct to 3 decimal places.

Question 9 **(50 marks)**

Oscar is taking some measurements and is using trigonometry to work out some angles, distances, and areas.

First, Oscar takes measurements of two adjacent triangular fields, **Field 1** (ABC) and **Field 2** (BDC), as shown in the diagram below (not to scale).
B lies on the line AD. $|AB| = 30$ m, $|BD| = 10$ m, $|AC| = 35$ m, and $|\angle CAD| = 50°$.

Note: the angle ABC is **not** a right angle.



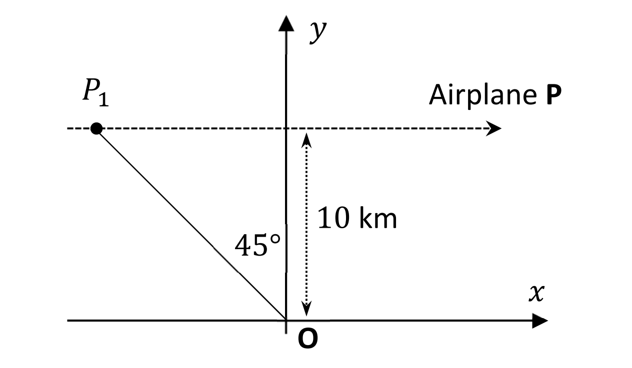
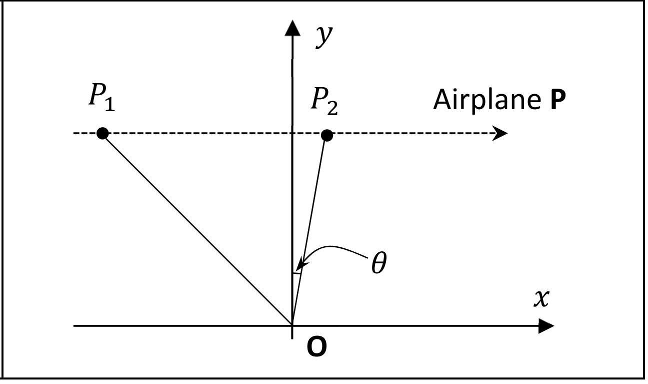
(a) Find the area of **Field 1** and, hence, find the area of **Field 2**.
Give each answer correct to the nearest m^2.

Area of **Field 1** = ________________ Area of **Field 2** = ________________

(b) Find the length of the perimeter of **Field 1**.
Give your answer correct to the nearest metre.

This question continues on the next page.

Oscar is also watching an airplane, **P**, fly directly over his head. He is standing at the point **O** in the diagrams below. The x-axis is the horizontal ground and the y-axis runs vertically up from Oscar. The airplane **P** is flying at a constant height of 10 km above the ground, as shown in the diagrams.

Diagram 1	**Diagram 2**
	

(c) Sound travels at a speed of roughly 343 metres per second in air.

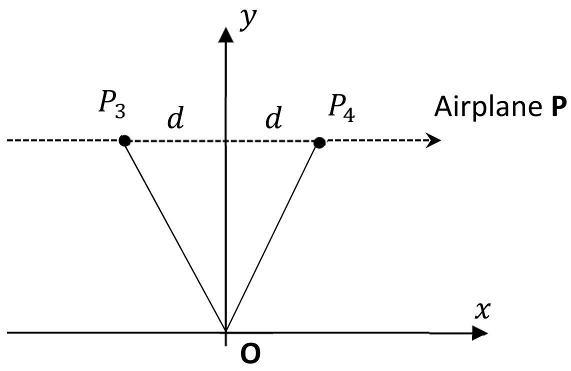
(i) As the airplane flies, its engine makes noise. It takes some time for this sound to reach Oscar. Use the information in **Diagram 1** to show that it takes 41 seconds for the sound the airplane makes at P_1 to reach Oscar, correct to the nearest second.

(ii) The airplane **P** is flying at a constant speed of 255 metres per second. By the time Oscar hears the sound the airplane made at the point P_1, the airplane has flown on to the point P_2, as shown in **Diagram 2** above.

Work out the size of the angle marked θ in **Diagram 2**, correct to 1 decimal place.

(d) P_3 and P_4 are two other points on the flightpath of airplane **P**.

By the time Oscar hears the sound the airplane made at the point P_3 , the airplane has flown on to the point P_4 , as shown in the diagram below (not to scale).
P_3 and P_4 are both a distance of d km from the y-axis.



(i) Explain briefly why the following equation holds:

$$\frac{\sqrt{100 + d^2}}{0{\cdot}343} = \frac{2d}{0{\cdot}255}$$

(ii) Solve the equation above to find the value of d, correct to 1 decimal place.

Question 10 **(50 marks)**

(a) In an athletics competition, there were a number of heats of the 1500 m race.
In the heats, the times that it took the runners to complete the 1500 m were approximately normally distributed, with a mean time of 225 seconds and a standard deviation of 12 seconds.

(i) Find the percentage of runners in these heats who took more than 240 seconds to run the 1500 m.

(ii) The 20% of runners with the fastest times qualified for the final.
Assuming the race times were normally distributed as described above, work out the time needed to qualify for the final, correct to the nearest second.

(b) Sally takes part in a number of different races in the competition.
The probability that she makes a false start in any given race is 5%.
Find the probability that she makes her first false start in her fourth race.
Give your answer correct to 4 decimal places.

(c) 20 relay teams took part in the competition. For any particular team, the probability that they drop the baton at some point during the competition is $0{\cdot}1$.

Find the probability that **at most 2** teams drop the baton during the competition.
Give your answer correct to 4 decimal places.

(d) 300 runners take part in a road race.
Each runner has a number, from 1 to 300 inclusive. No two runners have the same number.

Two runners are picked at random from the runners in this race.
Work out the probability that the sum of their numbers is 101.
Give your answer as a fraction in its simplest form.

This question continues on the next page.

(e) Sorcha ran two different marathons: the Windy Marathon and the Sunny Marathon. The table below gives some details on the finishing times for the two marathons. For each marathon, the finishing times of the runners were approximately normally distributed.

Sorcha's position in each marathon was based on her finishing time. Sorcha came 5265th in the Windy Marathon, so exactly 5264 of the 6000 runners had a finishing time that was less than Sorcha's.

Sorcha's finishing time for both marathons was the same.

Use this fact, and the details in the table, to estimate Sorcha's position in the Sunny Marathon. Show all of your working out.

	Mean finishing time (minutes)	**Standard deviation of finishing times** (minutes)	**Number of runners**	**Sorcha's position**
Windy Marathon	254	38	6000	5265th
Sunny Marathon	247	29	2000	

Page for extra work.
Label any extra work clearly with the question number and part.

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Leaving Certificate – Higher Level

Mathematics Paper 2

Monday 13 June

Morning 9:30 - 12:00