Worksheet 8- Paper 2

Q1. Caroline is suspended in a bouncer 25cm above the floor as shown. When she bounces up and down, the formula $b = 10\,sin\,45t^0$ models her bouncing height, b is measured in centimetres and t represents tenths of seconds.

(i) When the baby is bouncing, what is the maximum and minimum heights she will be above the floor?

(ii) Draw the graph for a duration of 2 seconds starting with t=0

(iii) What is the period of this function?

(iv) What vertical distance will the baby have travelled during these 2 seconds?

(v) If Caroline giggles happily when she changes direction, how many times will she giggle during the 2 seconds?

Q2. The area of a sector of a circle is 27cm^2, radius=6 cm. Find, in radians, the measure of the angle in the sector.

Q3. Find all the solutions of the equation $15\,sin^2\,x - 4\,cos\,x - 11 = 0\ , 0 \leq x \leq 360^0$ Give answers correct to the nearest degree.

Q4.

(i) Prove the formula $cos(A + B) = cos\,A\,cosB - sin\,A\,sin\,B$

(ii) Show that $\frac{1-tan^2\,A}{1+tan^2\,A} = cos\,2\,A$

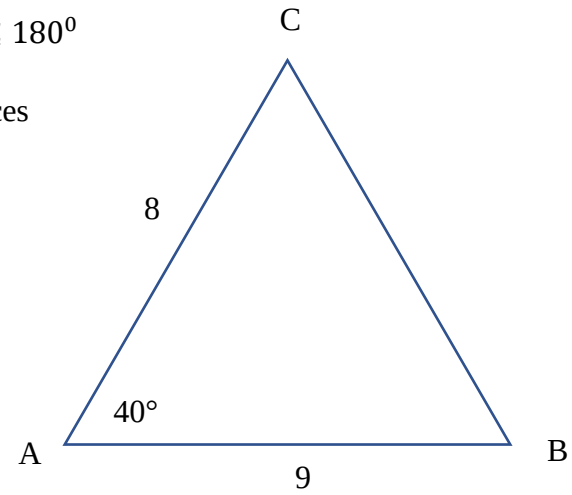
Q5. Find the value of k for which $sin\,75 - sin\,15 = \frac{1}{\sqrt{k}}, k \in \mathbb{N}$

Q6.

(i) Express $sin\,5x - sin\,x$ as a product of sine and cosine

(ii) Hence, solve $sin\,5x - sin\,x = 0, 0 \leq x \leq 180^0$

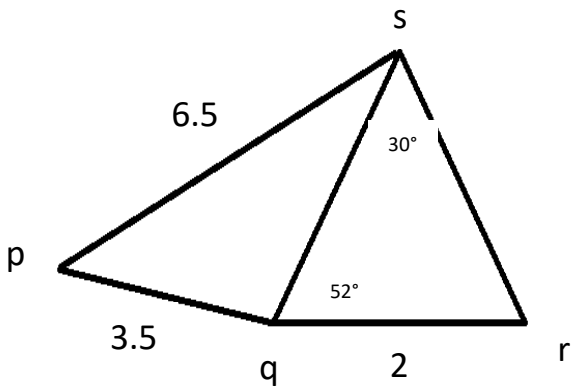
Q7. Find the area ΔABC, correct to 2 decimal places



Q8. Find

(i) $|qs|$ correct to two decimal places

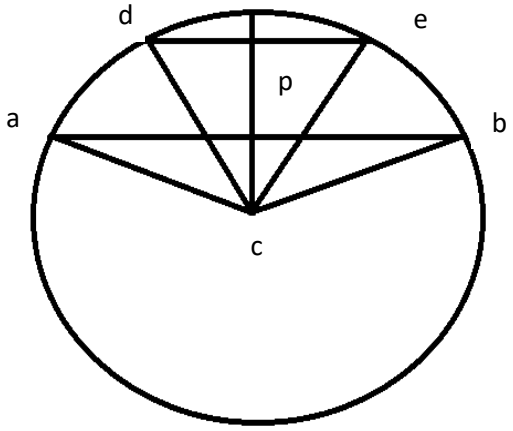
(ii) $|< pqs|$, correct to the nearest degree



Q9. Prove $tan(A+B) = \frac{tan\,A + tan\,B}{1 - tan\,A\,tan\,B}$. Hence find $tan\,75$ in the form of $p \pm \sqrt{q}, \quad q, p \in \mathbb{N}$

Q10. If $tanA = \frac{1}{2}$, find $tan2A$, without evaluating A, A is acute.

tan $(2A + B) = \frac{63}{26}$, find $tanB$ in the form $\frac{a}{b}, \; a, b \in \mathbb{N}$

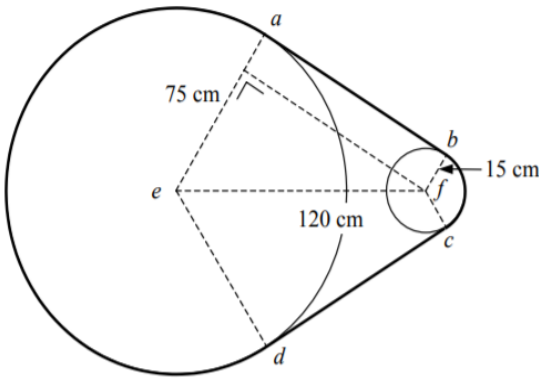


Q11. [ab] and [de] are two parallel chords of a circle with centre c and radius length r. $cp \perp de, |< acb| = 4\beta$ and $|< dce| = 2\beta$, where β is in radian measure, $\beta \neq 0$.

(i) If the area of the triangle abc = the area of the triangle dce, show that $\beta = \frac{\pi}{6}$.

(ii) Calculate the value of r if $(|ab|)^2 + (|de|)^2 = 24$ and give your answer as a surd.

Q12. A chain passes around two circular wheels as shown. One wheel as radius 75 cm and the other has radius 15 cm. The centres, e and f, of the wheels are 120 cm apart. The chain consists of a common tangent [ab], the minor arc bc, the common tangent [cd] and the major arc da.



(i) Find the measure of <aef.

(ii) Find $|ab|$ in surd form

(iii) Find the length of the chain, giving your answer in the form $k\pi + l\sqrt{3}$ where $k, l \in \mathbb{Z}$

Q13. A cylindrical shaped tin of height h and radius r has a volume of $98\pi\ cm^3$

(i) Show that $h = \frac{98}{r^2}$

(ii) Show that the total surface area is $S = 2\pi\left[\frac{r^3+98}{r}\right]$

(iii) Find the radius of the tin that will minimise the amount of aluminium required to manufacture the tin, correct to one decimal place.

(iv) Hence, or otherwise, find the minimum total surface area of the tin, correct to 2 decimal places.

Q14.

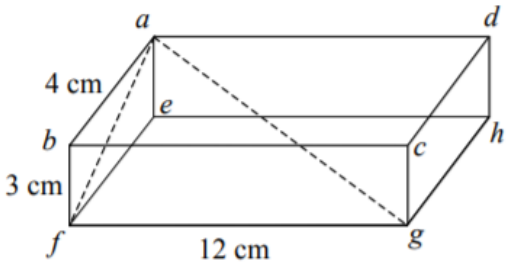
(a) Prove that $\cos^2 A + \sin^2 A = 1$, where $0^\circ \leq A \leq 90^\circ$.

(b) **(i)** Show that $(\cos x + \sin x)^2 + (\cos x - \sin x)^2$ simplifies to a constant.

(ii) Express $1 - (\cos x - \sin x)^2$ in the form $a \sin bx$, where $a, b \in \mathbf{Z}$.

(c) The diagram shows a rectangular box.
Rectangle *abcd* is the top of the box and rectangle *efgh* is the base of the box.

$|ab| = 4$ cm, $|bf| = 3$ cm and $|fg| = 12$ cm.



(i) Find $|af|$.

(ii) Find $|ag|$.

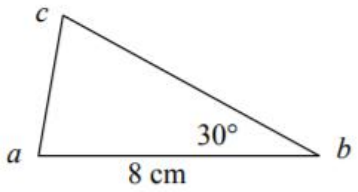
(iii) Find the measure of the acute angle between $[ag]$ and $[df]$.
Give your answer correct to the nearest degree.

Q15.

(a) The area of triangle abc is 12 cm^2.

$|ab| = 8$ cm and $|\angle abc| = 30°$.

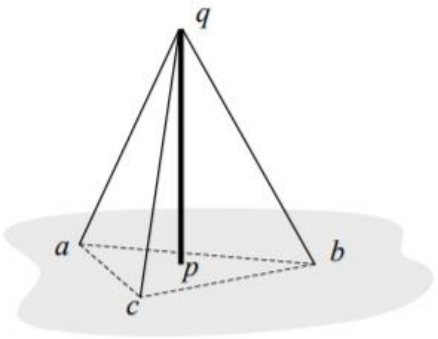
Find $|bc|$.



(b) **(i)** Prove that $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$.

(ii) Hence, or otherwise, prove that $\tan 22\frac{1}{2}° = \sqrt{2} - 1$.

(c) A vertical radio mast $[pq]$ stands on flat horizontal ground. It is supported by three cables that join the top of the mast, q, to the points a, b and c on the ground. The foot of the mast, p, lies inside the triangle abc.



Each cable is 52 m long and the mast is 48 m high.

(i) Find the (common) distance from p to each of the points a, b and c.

(ii) Given that $|ac| = 38$ m and $|ab| = 34$ m, find $|bc|$ correct to one decimal place.

Q16.

$f(x) = -3 + 4\sin 2x$

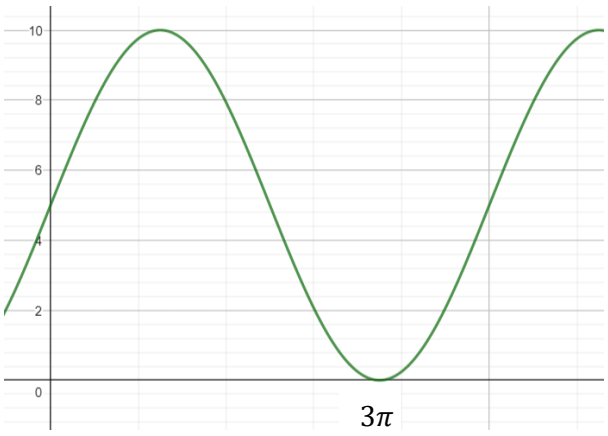
(i) State the period and the range of the graph

(ii) What is the equation of the midline?

(iii) Without, constructing a table of values draw the graph f(x) for $-\pi \leq x \leq \pi$

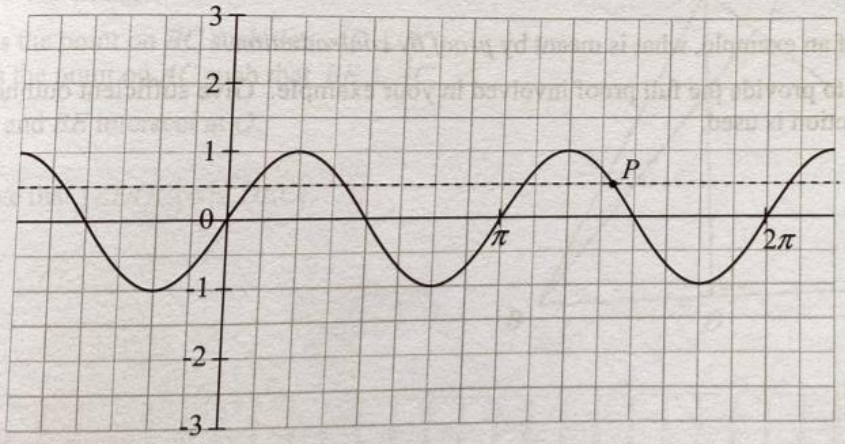
Q17. $g(x) = a \pm b\sin cx$

Find the values of a, b, and c.



Q18.

The diagram below shows the graph of the function $f: x \mapsto \sin 2x$. The line $2y = 1$ is also shown.



(a) On the same diagram above, sketch the graphs of $g: x \mapsto \sin x$ and $h: x \mapsto 3\sin 2x$. Indicate clearly which is g and which is h.

(b) Find the co-ordinates of the point P in the diagram.

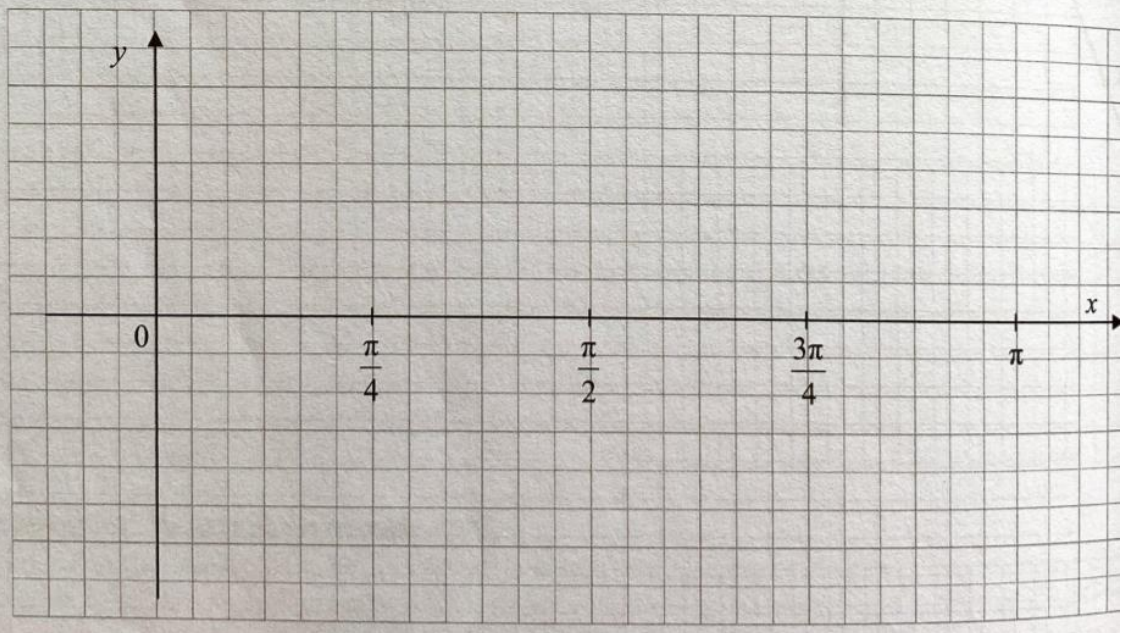
Q19.

The function $f: x \mapsto 3\sin(2x)$ is defined for $x \in \mathbb{R}$.

(a) Complete the table below

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
$2x$					
$\sin(2x)$					
$3\sin(2x)$					

(b) Draw the graph of $y = f(x)$ in the domain $0 \leq x \leq \pi,\ x \in \mathbb{R}$.



(c) Write down the range and the period of f.

Range = _____ Period = _____