<u>Worksheet 6- Paper 1</u>

Financial Maths, Patterns and Binomial Theorem

Q1. Most lottery games in the USA allow winners of the jackpot to choose between two forms of the prize: an annual-payments option or a cash-value option. In the case of the New York Lotto, there are 26 annual payments in the annual-payment option, with the first payment immediately, and the last payment in 25 years' time. The payments increase by 4% each year. The amount advertised as the jackpot prize is the total amount of these 26 payments. The cash-value option pays a smaller amount than this.

(a) If the amount of the first annual payment is A, write down, in terms of A, the amount of the second, third, fourth and 26th payments.

1st payment (now): A

2nd payment: __________

3rd payment: __________

4th payment:___________

26th payment: ___________

(b) The 26 payments form a geometric series. Use this fact to express the advertised jackpot prize in terms of A.

(c) Find, correct to the nearest dollar, the value of A that corresponds to an advertised jackpot prize of $21·5 million.

(d) A winner who chooses the cash-value option receives, immediately, the total of the present values of the 26 annual payments. The interest rate used for the present-value calculations is 4·78%. We want to find the cash value of the prize referred to in part (c).

(i) Complete the table below to show the actual amount and the present value of each of the first three annual payments.

Payment number	Time to payment (years)	Actual Amount	Present Value
1	0		
2	1		
3	2		

(ii) Write down, in terms of n, an expression for the present value of the nth annual payment.

(iii) Find the amount of prize money payable under the cash-value option. That is, find the total of the present values of the 26 annual payments. Give your answer in millions, correct to one decimal place.

(e) The jackpot described in parts (c) and (d) above was won by an Irish woman earlier this year. She chose the cash-value option. After tax, she received $7·9 million. What percentage of tax was charged on her winnings?

Q2.

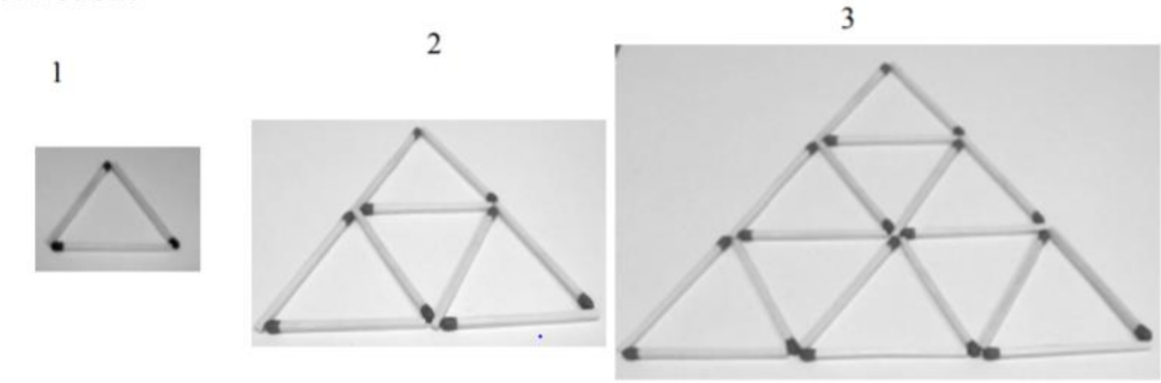
a) Niamh has saved to buy a car. She saved an equal amount at the beginning of each month in an account that earned an annual equivalent rate (AER) of 4%.
   (i) Show that the rate of interest, compounded monthly, which is equivalent to an AER of 4% is 0.327%, correct to 3 decimal places.
   (ii) Niamh has €15000 in the account at the end of the 36 months. How much has she saved each month, correct to the nearest euro?
b) Conall borrowed to buy a car. He borrowed €15000 at a monthly interest rate of 0.866%. He made 36 equal monthly payments to repay the entire loan. How much, to the nearest euro was each of his monthly payments?
c) Verify your answer using the Amortization Formula

Q3. Prove the Amortization Formula

$$A = \frac{Pi(1+i)^t}{(1+i)^t - 1}$$

Q4.

Shapes in the form of small equilateral triangles can be made using matchsticks of equal length. These shapes can be put together into patterns. The beginning of a sequence of these patterns is shown below.



(a)

(i) Draw the fourth pattern in the sequence.

(ii) The table below shows the number of small triangles in each pattern and the number of matchsticks needed to create each pattern. Complete the table

Pattern	1^{st}	2^{nd}	3^{rd}	4^{th}
Number of small triangles	1		9	
Number of matchsticks	3	9		

(b) Write an expression in n for the number of triangles in the n^{th} pattern in the sequence.

(c) Find an expression, in n, for the number of matchsticks needed to turn the $(n-1)^{th}$ pattern into the n^{th} pattern.

(d) The number of matchsticks in the n^{th} pattern in the sequence can be represented by the function $U_n = an^2 + bn$ where $a, b \in \mathbb{Q}$ and $n \in \mathbb{N}$. Find the value of a and the value of b.

(e) One of the patterns in the sequence has 4134 matchsticks. How many small triangles are in that pattern?

Q5. Use the Binomial Theorem to expand $(1 + 3x)^5$

Q6. Find the middle term in the expansion of $(2x + 1)^4$

Q7. Find the middle term in the expansion of $\left(x^2 - \frac{4}{x}\right)^6$

Q8. The coefficient of y^2 in the expansion of $(1 + ky)^4$ is 54. Find the values of k.

Q9. Write out the first 4 terms in the expansion of $(1 + 3x)^{10}$ and hence evaluate $(1.006)^{10}$.

Q10. $U_{n+1} = \frac{U_n + 3}{2}$

(i) If $U_1 = 7$, write out the values of $U_2,\ U_3, U_{4,} U_5$ *and* U_6

(ii) Do you think this sequence has a limit?

(iii) If a sequence appears to get closer and closer to a specific value, it is said to be______

Q11. Find the greatest value of *x*, $0 \leq x \leq {}^{\pi}/_{2}$ for which

$\left(1 + sin^2 x + sin^4 x + \cdots + sin^{(2n-2)} x \dots \dots \dots\right) \leq 4$

Q12. a, b, c and d are in geometric sequence. Prove that $a^2 - b^2 - c^2 + d^2 \geq 0$

Q13. $T_n = an^2 + bn + c$

(i) Find T_1, T_2, T_3, T_4 and T_5

(ii) Hence, find the first difference and the common second difference

Q14. $T_n = an^3 + bn^2 + cn + d$

(i) Find T_1, T_2, T_3, T_4 and T_5

(ii) Hence, find common third difference of a cubic equation

Q15. Find an algebraic expression to represent

a) -1, 3, 15, 35, 63

b) 3, -1, -1, 9, 35

Q16. $f(x) = ax + b, x \geq -2$ represents the number pattern 1, 3, 5, 7, 9…

Find the value of a and b.