Worksheet 5- Paper 1

Q1. $z_1 = s + 8i$ $\quad z_2 = t + 8i, \quad s, t \in \mathbb{R} \quad i^2 = -1$

(i) $|z_1| = 10$, find 2 possible values of s

(ii) $arg(z_2) = \frac{3\pi}{4}$, find the value of t

Q2. $z_1 = a + bi$ $\quad z_2 = c + di$

Show that $\overline{z_1 + z_2} = \overline{z_1} + \bar{z}_2$

Q3. $z = \frac{5}{2+i} - 1$

(i) Express z in the form of $x + iy$ and plot it on an argand diagram

(ii) Use De Moivre's Theorem to evaluate z^6

Q4. $z = -1 + i$

(i) Use De Moivre's Theorem to find z^5 and z^9

(ii) Show that $z^5 + z^9 = 12z$

Q5. Solve the equation;

$a)\ (a + bi)^2 = 15 + 8i \quad a, b \in \mathbb{R}$

$b)\ Hence, solve\ \ iz^2 + (2 - 3i)z + (-5 + 5i) = 0$

Q6.

(i) Express $-8 - 8\sqrt{3}i$ in the form of $r(cos\,\theta + i\,sin\,\theta)$

(ii) Find $\left(-8 - 8\sqrt{3}i\right)^3$

(iii) Find the four complex numbers for which $z^4 = -8 - 8\sqrt{3}i$. Give your answers in the form of $a + bi$, with a and b fully evaluated

Q7. Find the real numbers p and q such that

$2(p + iq) + i(p - iq) = 5 + i, \qquad i^2 = -1$

Q8. $z_1 = cos\frac{4\pi}{3} + i\,sin\frac{4\pi}{3}$, $z_2 = cos\frac{\pi}{3} + i\,sin\frac{\pi}{3}$.

Evaluate $z_1 z_2$, giving your answer in the form of $x + iy$.

Q9. $w_1 = a + ib \qquad w_2 = c + id$

Prove that $(\overline{w_1 w_2}) = (\overline{w_1})(\overline{w_2})$

Q10. Simplify $\left(\frac{-2+3i}{3+2i}\right)$ and hence evaluate $\left(\frac{-2+3i}{3+2i}\right)^9, \quad i^2 = -1$

Q11. Use De Moivre's Theorem to prove that $cos\,3\theta = 4\,cos^3\,\theta - 3\,cos\,\theta$

Q12. Use De Moivre's Theorem to express $\left(-\sqrt{3} - i\right)^{10}$ in the form of $2^n\left(1 - i\sqrt{k}\right), n, k \in N$

Q13.

(i) Find the quadratic equation which has roots $3 + i$ and $3 - i$

(ii) $P(z) = z^3 - kz^2 + 22z - 20, \quad k \in \mathbb{R}$

$3 + i$ is a root of $P(z) = 0$. Find the value of k and the other 2 roots

Q14. $\sqrt{5}|w| + iw = 3 + i$.

Solve for w, leaving your answers in the form of $u + iv, \qquad u, v \in \mathbb{R}$