Worksheet 4- Paper 1

Q1.

a) Use first principles to differentiate $f(x) = 3 - 4x - 2x^2$

b) $f(x) = e^{2x}, find\ f'(x)\ and\ f''(x)$

c) $f(x) = 5\sin 2x\ ,find\ f'(x)\ and\ f''(x)$

Q2. $y = e^{2x}$ and $y + \frac{dy}{dx} + \frac{d^2y}{dx^2} = ke^{2x}$. Find k.

Q3. $f(x) = 3\cos(2x + 5)$

Show that $f''(x) + 4f(x) = 0$

Q4. $f(x) = \frac{2}{1-2x}$

a) Show that f(x) is increasing and has no point of inflection

b) $y = x + c$ is a tangent to f(x). Find 2 possible values for c, c is a real number

Q5. $f(x) = \frac{x}{x-3}\ , x \in \mathbb{R}, x \neq 3$

Find f'(x) and f''(2)

Q6. Use calculus to find the slopes of the tangents to the circle $x^2 + y^2 = 25$ at $x = -4$. Hence, find the equation of the tangents.

Q7. The volume of a cube is increasing at the rate of $12\ cm^3/second$. Find the rate of increase of l, the length of the edge of the cube when the volume is $125\ cm^3$.

Q8. A football is travelling along a path y, where $y = x - \frac{x^2}{40}, x \geq 0$.

If $\frac{dx}{dt} = 10\sqrt{2}$, find $\frac{dy}{dt}$ when $x = 10$.

Q9. $x + y = 100$

(i)   Prove that the product P=xy is a maximum when x=y.

(ii)   Find the maximum value of P.

Q10. Find the range of values of x for which $f(x) = x^3 + 3x^2 - 9x$ is decreasing

Q11. $f(x) = 2x^2 - 3x + 2$ $g(x) = x^2 + x + 7$

a) Find the co-ordinates of the point of intersection of f(x) and g(x)

b) Find the area of the region enclosed between the two curves

Q12. v=velocity

$v = 5t^2 + 8t - 1$ Find;

a) The distance travelled in terms of t

b) The acceleration after 8 seconds

Q13. $f''(x) = 15x - 2$,

$f'(x) = 23$ when x=2 and y=19. Find $f(x)$.

Q14. Find the constant of integration if $\int (6t^2 + 12t + 5)\, dt = 9$ when t=-2

Q15. Show that $\frac{1}{3} \int_{-2}^{26} \frac{dx}{x+6} = ln\, 2$

Q16. Find the area enclosed by $y = x^2 + 1$ and y=5

Q17. Find the mean value of $y = \sin x, 0 \le x \le \frac{\pi}{2}$

Q18. $y = \frac{\cos x + \sin x}{\cos x - \sin x}$

(i) Find $\frac{dy}{dx}$

(ii) Show that $\frac{dy}{dx} = 1 + y^2$

Q19. Find $\int (\sin 2x + e^{4x}) \, dx$

Q20. $y = 12x^3 - 48x^2 + 36x$

 (i) Find the co-ordinates of the 3 points where the curve crosses the x-axis

 (ii) Sketch the curve

 (iii) Calculate the total area of the region enclosed between the curve and the x-axis

Q21. The equation of a curve is $y = \frac{2}{x-3}$. Prove that no two tangents to the curve are perpendicular to each other.

Q22. Differentiate $\sin(3x^2 - x)$ with respect to x

Q23. The equation of a curve is $x^2 - y^2 = 25$. Find $\frac{dy}{dx}$ in terms of x and y.

Q24. Find $\int \left(6x + 3 + \frac{1}{x^2}\right) dx$

Q25. Evaluate $\int_{-\pi/4}^{\pi/4} \sin 3x \sin x \, dx$

Q26. $f(x) = 2x^3 + 3x^2 + bx + c$ has a local maximum at x=-2.

(i)   Find the value of b

(ii)   Find the range of values of c for which $f(x) = 0$ has 3 distinct real roots

Q27. Differentiate $2x + \sin 2x$

Q28. Find $\int (2x + \cos 3x)dx$

Q29. $f(x) = \log_e 3x - 3x$

(i)   Show that $\left(\frac{1}{3}, -1\right)$ is a local maximum point of $f(x)$

(ii)   Deduce that the graph of f(x) does not intersect the x-axis

Q30. The line $2x - y - 10 = 0$ is a tangent to the curve $y = x^2 - 9$. The shaded region is bounded by the curve, the line and the x-axis. Calculate the area of this region.