Algebra, Sequences and Series and Logs

Worksheet 3-Paper 1

Q.1 Find the sum of the first 100 natural numbers

Q.2 Using a formula in the maths tables, find the sum of the first n natural numbers and verify your answer by Induction

Q.3 Write $0.\dot{3}\dot{6}$ in the form of $\frac{a}{b}$, where a, b $\varepsilon\, \mathbb{N}$

Q.4 What is meant by an irrational number

Q.5 Simplify a) $\frac{x^3 - 8y^3}{x - 2y}$ b) $\frac{x^{3/2} - x^{-1/2}}{x^{1/2} - x^{-1/2}}$

Q.6 State the factor theorem and explain how you would use it to solve a cubic equation

Q.7 If $x^2 + 2ax + a^2$ is a factor of $x^3 + 6kx + k$, show that $a^2 = -2k$

Q.8 Sketch e^x and e^{-x} using the same axes and scale

Q.9 Solve $log_2(x + 2) + log_2 x = 0, x > 0.$ Leave your answer to three decimal places.

Q.10 Simplify $\frac{3}{\sqrt{2}-1}$

Q11 Find the sum of all the natural numbers from 101 to 200.

Q12 Prove, De Moivre Theorem, by Induction.

Q13 Prove that $a + ar + ar^2 + \cdots + ar^{n-1} = \frac{a(1-r^n)}{1-r}$

Q14 Prove, by induction that the sum of the first n odd natural numbers is n^2.

Q15 Write $10.\dot{5}\dot{1}$ in the form of $\frac{a}{b}$, where a and b are natural numbers.

Q16 Evaluate $\lim_{n\to\infty} \frac{\sqrt{-1+2n^2}}{1+n}$

Q17 $f(x) = 2x - 1$ and $g(x) = 3x^2$

   (i)   Find $gf(x)$

(ii)   Verify your answer using $x = 2$

Q18 Evaluate

(i) $\lim\limits_{x\to 7} \dfrac{x^2-5x-14}{x-7}$

(ii) $\lim\limits_{n\to\infty} \dfrac{n^2+9}{2n^2+9n}$

Q19 $g(x) = x^2 - 5$, x is a real number

(i)   Find $g^{-1}(x)$

(ii)   Show that $gg^{-1}(x) = x$

Q20 Simplify $\dfrac{8x^3-27}{2x-3}$

Q21 Find s_∞ of $\displaystyle\sum_{n=0}^{\infty} \left(\dfrac{5}{2x-1}\right)^n$

Q22 Prove, by Induction $x + x^2 + x^3 + \cdots x^n = \dfrac{x(x^n-1)}{x-1}, x \neq 1$

Q23 Prove, by Induction $3^n > n^2, n > 1$, n is a natural number

Q24

(i) a, ar, ar^2 ... is a geometric sequence. Find an expression for S_{2k}

(ii) b, br^2, br^4 ... is another geometric sequence. Find an expression for S_k

(iii)   If $S_{2k}=S_k$, show that $r = \dfrac{b}{a} - 1$

Q25 Write $-x^2 - 12x + 5$ in the form of $p - (x + a)^2, a, p \text{ are real numbers}$

Q26 By completing the square find g^{-1} of $g(x) = x^2 + 6x + 1$

Q27 $f(x) = 4x^{-p}$, find the value of p for which $f(x^{-1}) = f(x)$, x is a real number

Q28 Find the sum of the infinite series,

$$\sum_{n=0}^{\infty} \left(\dfrac{3x}{3x + 4}\right)^n, x > 0, x \in \mathbb{R}$$

Q29 Using the same axes and scales, sketch the curve $g(x) = 2^x$ and $f(x) = log_2 x$

Q30

    (i)   Use the algebra to find the maximum value of $y = -x^2 - 12x + 5$

    (ii)   Given that $f(x) = -2x^2 - 16x + k$ has the same maximum value as y in (i), find

           the value of k, k is a real number.